

A Fuzzy Mathematical Linear Programming for Aggregated Production Planning: A Case Study for Furniture Company

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Abstract

Fuzzy programming is used in situations where some of the parameters of a mathematical model are described by fuzzy variables instead of deterministic variables. In modern businesses, production planning is needed to minimize the loss of material, machine time and labor force in order to enable the business to run at an economic level. The goal of production planning is to compare effectively the production facilities and present resources with each other and to allocate resources to the related productions according the periods. In this study, we proposed a fuzzy linear programming (FLP) for aggregated production planning (APP) of a furniture company. We discussed the solution of FLP problem with the help of multi objective constrained linear programming problem. Based on the suggested production plan, it is possible to get new results by changing product numbers, types and number of planning periods.

Keywords: Fuzzy Linear Programming, Aggregated Production Planning, Furniture

1. Introduction

Production planning is necessary for modern business for workload of production planning, coordination of domestic business activities, improvement of relations and dependancy among business firms, expanding of procurement and distribution in a wide area, increasing of price competition and service quality, aiming of providing running of business firm, materials, minimizing of working labour. In the real-world multi-period production planning (MPPP) problems, the parameters must be estimated and they are -frequently given by interval estimates. But for almost all current production planning models, these interval estimates must be translated into single numbers. This kind of translation often results in errors and in the loss of a considerable amount of information. Today's competitive environment, production planning is in key position to firms' success. Therefore, decision makers have to use scientific methods at production planning. One of the inputs to the production planning is the sales forecasts; but these need to be stated on the basis of shipments (not bookings) so the inventory projections match physical inventories and so demands on manufacturing are expressed correctly with respect to time.

In production and operations management, one of the most important functions is APP. Some of the other types of production planning are master production schedule, capacity requirements planning and material requirements planning. All these functions depend on the APP in a hierarchical way [1].

Short, medium and long term production planning horizons are considered as three production planning type. In this research medium term production planning horizon is chosen. Medium term may be for as short as three months or as long as twelve months. There are a lot of solving

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methods of an APP problem in literature. One of the widely used decision making method is linear programming. In linear programming applications the parameters that are using is constant but real words problems are very complex and hard to determine parameters accurately. Day by day furniture industry grown and now impossible to determine factors such as the customer demand, production costs and inventory costs which contains some uncertainties. Many researchers try to find solutions for this issue and they realized using fuzzy data may play a significant positive role in overcoming the difficulties. Nowadays many researchers aware of fuzzy programming is more effective instead of linear programming. Rommelfanger [2] mention in order to reduce information cost and at the same time avoid unrealistic modeling, the use of FLP can be recommended. In fuzzy logic, the focus of the problem is not “either exists or not” but “to what extend it exists”. In FLP goals and constraints can involve some violations. The model may not have a goal. The information about the goal may only be a verbal statement The FLP model which is an extended mode of the linear programming approach applied in classical logic is the method which ensures the optimal decision in situations where lack of information and uncertainties exists. The goal here is to make optimal decision by finding faster and more flexible solutions.

The aim of this study is to develop a fuzzy multi-objective linear programming for solving this production planning decision. The model objective is maximizing to profit that is determined by decision makers. Decision makers also determined to “satisfaction level” for profit goal.

The organization of this paper is as follows. After this introduction, literature review given. A Fuzzy linear programming model is formulated to solve the production planning problem in Section 3. In Section 4, a set of data from the furniture fabric- is used to test the effectiveness and the efficiency of the proposed model. Our conclusions are given in the final section.

2. Fuzzy linear programming

Zadeh [3] first time define fuzzy sets. In his article, he reveals the fuzzy set theory which has uncertain bound. Fuzzy set theory efficient for define uncertainty. Zimmermann offer to use fuzzy sets at multi-objective optimization problems. He studied duality analyze in fuzzy programming. Following Zimmermann, Bellman and Zadeh used linear memberships function instead of traditional linear programming. After that they developed FLP method to solve aggregated production planning problems. Lee [4], offered FLP model for APP problem, but in his model, he only considers single objective function for total cost. This assumption is sufficient for APP. Wang and Fang [5], introduced fuzzy environment and made an application to APP problem. In his research product price, unit cost to subcontract, work force level, production capacity and market demands are all fuzzy in nature. Sengül [6] used mixed integer programming for production planning. According to the literature review, there are some papers on fuzzy sets for production planning in furniture companies. The used parameters on production planning in furniture industries are hard to determine as deterministic values. Hence, the fuzzy parameter can be more efficient and robust for linear programming. In this paper, we proposed FLP model to evaluate production planning in healthy manner.

Linear programming is one of the most common technic for production planning. The decision space is defined by constraints and an objective function. It can be applied to many problems but it fails to deals with imprecise data. Thus decision making occurs under certainty. In linear fuzzy programming, optimization occurs under uncertainty. For fuzzy linear programming we will need to depart from the classical assumption of linear programming. We can transform linear programming problem to fuzzy linear problem with Zimermans method [7].

Linear Programming model

$$\text{Max } z \cong c_i x_i \tag{1}$$

$$\text{s.t. } a_i x_i < b_i \tag{2}$$

For treating fuzzy inequalities, Zimmermann proposed linear membership function as follows:

$$\text{Max } z = \lambda \tag{3}$$

$$c_i x_i > b_i - p_i (1 - \lambda) \tag{4}$$

$$a_i x_i < b_i + p_i (1 - \lambda) \tag{5}$$

$$\lambda \in [0, 1] \tag{6}$$

b_i is the i th element of b and for $i = 0, 1, \dots, m$, p_i is a subjectively chosen constant by the Decision makers expressing the limit of the admissible violation of the i th inequality.

3.1. Problem Statement and Assumptions

In this paper, some assumptions are taken into consideration for solving proposed FLP model in feasible. The regular production time and warehouse are considered as limited. The limited is determined by experts as maximum levels. The capacity of the machines for each period cannot exceed upper limits. The all raw materials are purchased in unlimited levels. The all objective functions and constraints are assumed as linear. In FLP model, the objective function has imprecise aspiration level. All product is manufactured in company and there is no subcontracting. The all products have a production flow and this assumption cannot be have omitted.

3.2. Model Formulation

$$\text{Max } = \lambda \tag{7}$$

$$\sum_{\varphi=29}^{34} ((P_{\varphi} Q_{\varphi}) - (C_{\varphi} Q_{\varphi})) \geq b - P_{\psi} (1 - \lambda) \tag{8}$$

$$\sum_{j=1}^{17} \sum_{i=1}^{146} (\mu_{\omega} X_{ij}) \leq \delta_{\omega} + p_{\psi} (1 - \lambda) \tag{9}$$

$$0 \leq \lambda \leq 1 \tag{10}$$

$$\forall (X_{i,j} = X_{i+1,j} = X_{i+2,j} = X_{i+3,j} = X_{i+4,j} = X_{i+5,j}) \tag{11}$$

- | | |
|--|---------------------------|
| P_{ψ} : tolerance | b : aspiration level |
| δ_{ω} : working time for workstation ω | X_i : products |
| Q_{φ} : Size of product | λ : objective |
| C_{φ} : i Cost of products | $\varphi = 29 \dots 34$; |
| P_{φ}, i : Sale price of products | $i = 1 \dots 145$; |
| $\psi = 1 \dots 17$; | $j = 1 \dots 17$; |
| μ_{ji} : duration of the processing of the product x_i in the work station j | |

3. A Case Study for Furniture Company

In this section, we present a mathematical formulation of furniture production planning model under uncertain environment. This research dealing in furniture production. The multi-product APP problem can be described as follows. Company's product range consist of six models of junior rooms. Juniou rooms consist of bookcase, desk, laundry, nightstand, bedstead. The purpose of the study is to determine the production sizes which will maximize the average profit of a factory which produces furniture products by using FLP model.

Junior room production is carried out by using 17 workstations. There are 6 different models of junior rooms in total. Products are sold as sets. In other words, when one model of junior room is sold, one bookcase, one desk, one laundry, one nightstand, one bedstead and one wardrobe of the same model will also be sold. In order to meet this condition, 16th equation is added to the model. Product names are shown in Table 1 as Q_{ij} . Product prices are shown as Q . The subindices of P and Q denote the same products. λ is the goal function of linear fuzzy programming. It takes values between 0-1. Value 1 means that achieving the goal is %100 percent in defined circumstances. In Table 1 there are explanations of the products.

Table 1. Products

Products	Products Name
Q29	Enerjik junior room
X12	Enerjik bookcase
X22	Enerjik desk
X31	Enerjik laundry
X42	Enerjik nightstand
X51	Enerjik Bedstead
X72	Enerjik wardrobe
Q30	Final junior room
X13	Final bookcase
X23	Final desk
X32	Final laundry
X44	Final nightstand
X53	Final bedstead
X74	Final wardrobe
Q31	Jasmin junior room
X14	Jasmin bookcase
X24	Jasmin desk
X33	Jasmin laundry
X45	Jasmin nightstand
X54	Jasmin bedstead
X65	Jasmin wardrobe
Q32	Liza junior room
X15	Liza bookcase
X25	Liza desk
X35	Liza laundry
X413	Liza nightstand
X512	Liza bedstead
X614	Liza wardrobe

$$\text{Max} = \lambda$$

(12)

$$(2330*Q29 + 1945*Q30 + 2220*Q31 + 2775*Q32 + 2065*Q33 + 1200*Q34) - (1320*Q29 + 995*Q30 + 1170*Q31 + 1218*Q32 + 1107*Q33 + 500*Q34) \geq 1130000 - 90000*(1 - \lambda) \tag{13}$$

$$0.759*x11 + 1.046*x12 + 1.046*x13 + 0.813*x14 + 1.075*x15 + 1*x16 + 0.297*x21 + 0.722*x22 + 0.562*x23 + 1.123*x24 + 0.921*x25 + 1.128*x26 + 1.102*x27 + 0.776*x31 + 0.776*x32 + 0.803*x33 + 1.040*x34 + 0.971*x35 + 0.989*x36 + 0.795*x37 + 0.184*x41 + 0.401*x42 + 0.832*x43 + 0.401*x44 + 0.345*x45 + 0.698*x46 + 0.728*x47 + 0.364*x48 + 0.699*x49 + 0.495*x51 + 0.90896*x52 + 0.97552*x53 + 1.137552*x54 + 1.547*x55 + 2.498*x56 + 2.5064*x57 + 0.428*x58 + 0.649*x59 + 1.765*x510 + 0.261*x511 + 0.43472*x512 + 1.859*x513 + 1.7*x517 + 1.88*x518 + 0.5767*x61 + 0.656656*x62 + 1.313*x63 + 1.89*x64 + 2.621*x65 + 2.751*x66 \leq 6804 + 756*(1 - \lambda); \tag{14}$$

$$\text{Profit} = (Q29*P29) + (Q30*P30) + (Q31*P31) + (Q32*P32) + (Q33*P33) + (Q34*P34) \tag{15}$$

$$\begin{aligned} X12 &= X22; X22 = X32; X32 = X42; X42 = X52; X52 = X62; X62 = Q29; \\ X13 &= X23; X23 = X33; X33 = X43; X43 = X53; X53 = X63; X63 = Q30; \\ X14 &= X24; X24 = X34; X34 = X44; X44 = X54; X54 = X64; X64 = Q31; \\ X15 &= X25; X25 = X35; X35 = X45; X45 = X55; X55 = X65; X65 = Q32; \\ X16 &= X26; X26 = X36; X36 = X46; X46 = X56; X56 = X66; X66 = Q33; \\ X11 &= X21; X21 = X31; X31 = X41; X41 = X51; X51 = X61; X61 = Q34; \end{aligned} \tag{16}$$

In this study, Twenty seven work stations were used in the production of junior rooms. Here only one of the work station’s technologic constraint is given. In this study the aimed profit level is determinated 1130000 TL. Moreover object function’s tolerance (p0) is determinate 90000 TL.

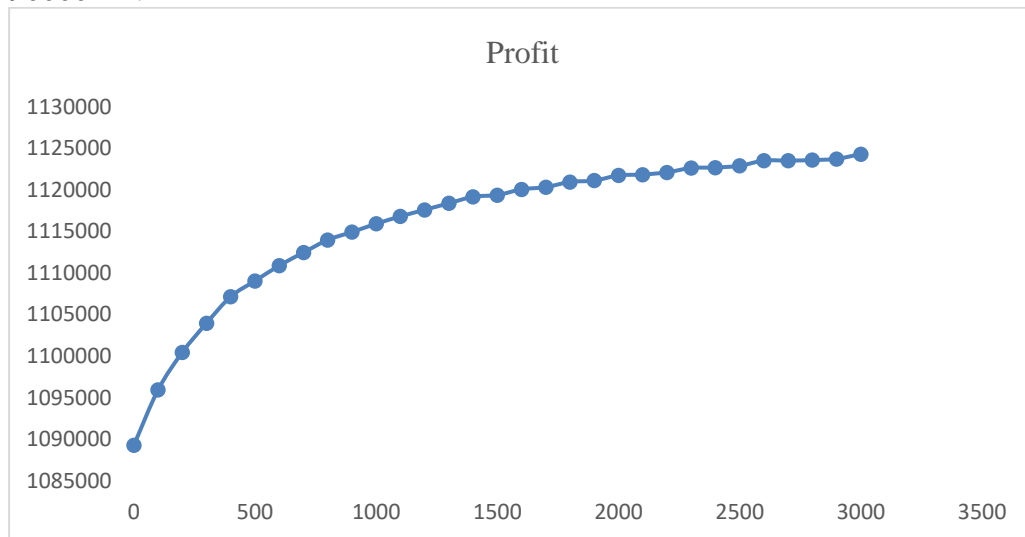


Figure 1. Variation of profit (TL) with respect to p1-p17

In Figure 1, the change of profit with the tolerance values from 17 work stations is to be seen. The tolerance value of seventeen work station was taken the same and the profit change was monitored by giving different values of tolerance from 0 to 3000. After having the results, the amount of profit increased as the tolerance values increased. The increase of tolerance values

between 0 and 500 has contributed to the rise of profit significantly. The cases, where the tolerance value is higher than 1500, did not affect the profit much.

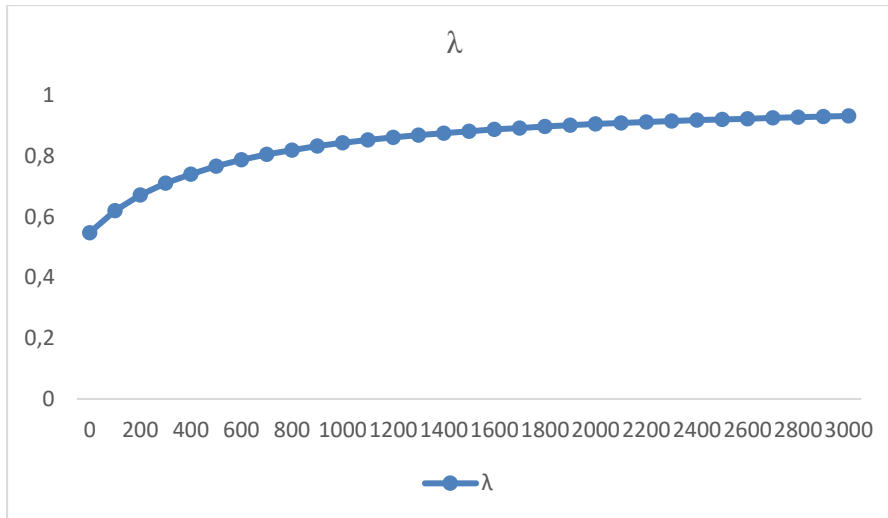


Figure 2. Variation of λ with respect to p1-p17

In Figure 2, the effect of change of tolerance to λ was investigated. As tolerance value increases, λ value also increases. A solution which provides the total satisfaction of fuzzy target and fuzzy limiters could not be found. For the tolerance value of 3000 of the work stations, the maximum membership function was found as 0.9318807. This shows that the fuzzy target and limiters were satisfied with the membership degree of 0.9318807. The expectation of profit in this case is 1124299 TL.

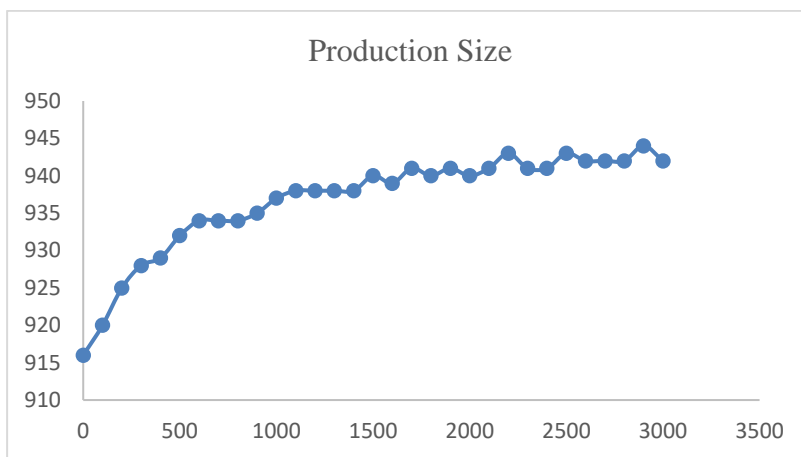


Figure 3. Variation of production size with respect to p1-p17

As the tolerance time of the workstations increases, the facility is in trend of producing more products in order to increase the profit. However, in some cases, the profit was increased by reducing the amount of product but choosing the products having more rate or profit.

4. Conclusions

We have presented and tested a generalized FLP model for production planning in furniture company. FLP model is a fast flexible and effective method for finding optimal solution in

cases where goal and constraints has some violations. As the solution the types of products and the sizes of production are determined which will maximize the profit for a period by carrying out time studies for the company.

The research will be continued in direction of further extension of the proposed production–distribution aggregate planning model by representing additional sources of uncertainty such as fuzzy lead times, fuzzy supplier reliabilit. One possible extension of the current model is to consider non-linear objective function. Non-linear programming (NLP) has advantages over linear programming (LP) as NLP considers certain factors such as economies of scale and economies of scope which LP tends to ignore.

6. References

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